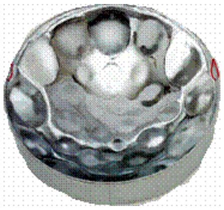
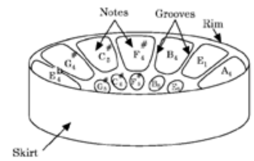
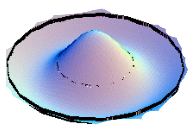


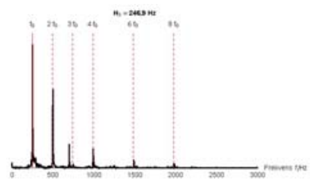


Olietøndens fysik

Hovedfagskollokvium af
Morten Gersborg-Hansen



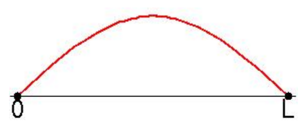
8. juli 2004



Simpelt system: 1D

Udsvinget af en snor
opfylder bølgeligningen:

$$\boxed{\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}} \qquad c^2 = \frac{F}{\rho}$$



Grænsebetingelse:

$$u(0) = u(L) = 0$$

Tilladte frekvenser:

$$\omega_n = \frac{n\pi c}{L} \quad \text{dvs.} \quad f_n = \frac{\omega_n}{2\pi} = n\frac{c}{2L}$$

Normalsvingninger:

$$u_n(x,t) = (a_n \sin\omega_n t + b_n \cos\omega_n t)\sin\frac{\omega_n x}{c}$$

Harmoniske:

$$f_n = nf_1$$

Generel løsning:

$$u(x,t) = \sum_n c_n u_n(x,t)$$

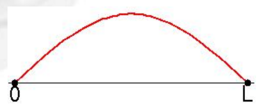
Ref.: [1], [7]

Normalsvingninger: 1D

Frekvens $\quad f_n = n f_1 \qquad f_1 = \dfrac{c}{2L}$

Grundtone

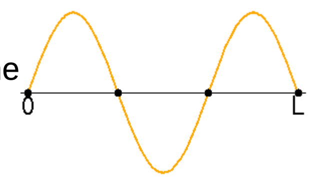
$n = 1$



(0)

2. overtone

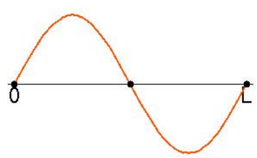
$n = 3$



(2)

1. overtone

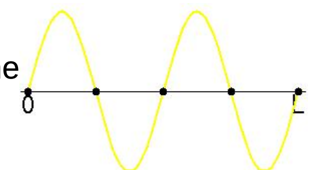
$n = 2$



(1)

3. overtone

$n = 4$



(3)

2D: Membransvingninger

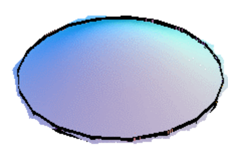
Forskydningen af en fleksibel membran opfylder 2D-bølgeligningen:

$$\boxed{\frac{\partial^2 u}{\partial t^2} = c^2 \nabla^2 u}$$

$$c^2 = \frac{T}{\rho}$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}$$



Seperation af de variable:

$$u(r,\theta,t) = F(r)G(\theta)H(t)$$

Grænsebetingelse:

$$u(R,\theta,t) = 0$$

Normalsvingninger:

$\sin n\theta$

$$u_{mn}(r,\theta,t) = (a_{mn}\cos ck_{mn}t + b_n \sin ck_{mn}t) J_n(k_{mn}r)\cos n\theta$$

$k_{mn} = \dfrac{\alpha_{mn}}{R} \qquad \alpha_{mn}$: m'te positive nulpunkt af $J_n(x)$

Ref.: [1], [7]

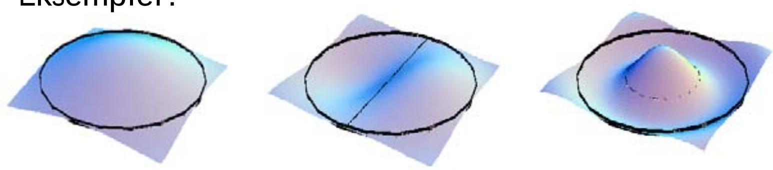
2D: Membransvingninger

$$u_{mn}(r,\theta,t) = (a_{mn}\cos ck_{mn}t + b_n \sin ck_{mn}t) J_n(k_{mn}r)\cos/\sin n\theta$$

Frekvens: $f_{(0,0)} = \frac{\alpha_{10}}{2\pi R}\sqrt{\frac{T}{\rho}} = \frac{2{,}405}{2\pi R}\sqrt{\frac{T}{\rho}}$

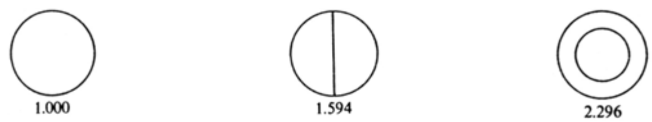
Indicering v. antal knudelinier som (r,θ): $(m\text{-}1,\ n)$

Eksempler:



(0,0) (0,1) (1,0)

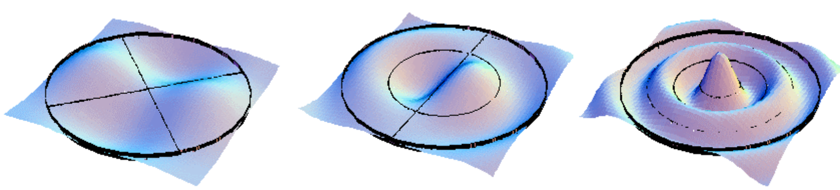
$\frac{f}{f_{(0,0)}}$



Ref.: [1]

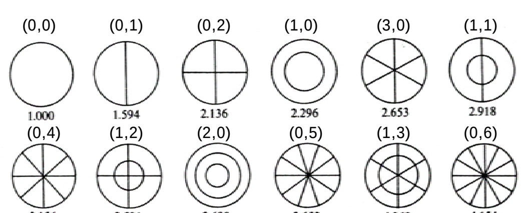
2D: Membransvingninger

Flere eksempler:



(0,2) (1,1) (3,0)

Udvalgte modes med $\frac{f}{f_{(0,0)}}$



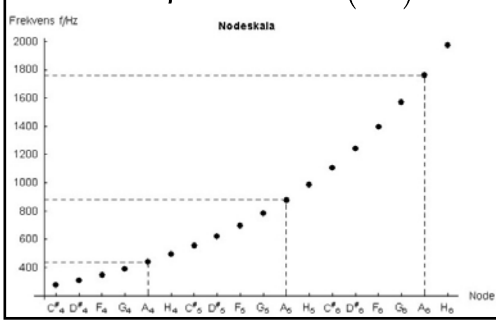
Ref.: [1]

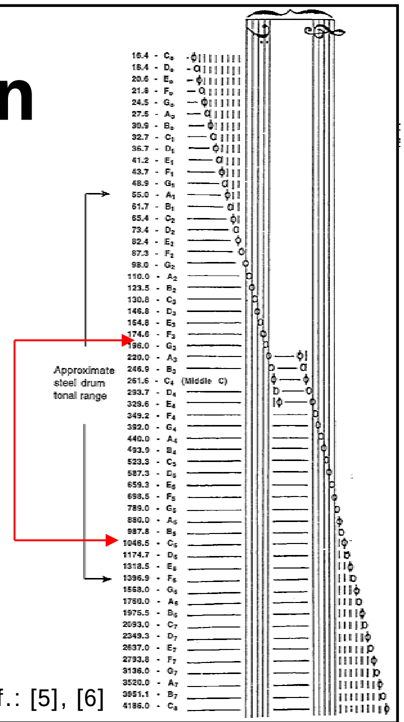
Nodeskalaen

Konstruktion:

- A_4 = 440 Hz (kammertonen)
- Oktav = dobbelt frekvens
- 12 halvtoner pr. oktav

dvs. $f = 440\,\mathrm{Hz} \cdot \left(\sqrt[12]{2}\right)^n$



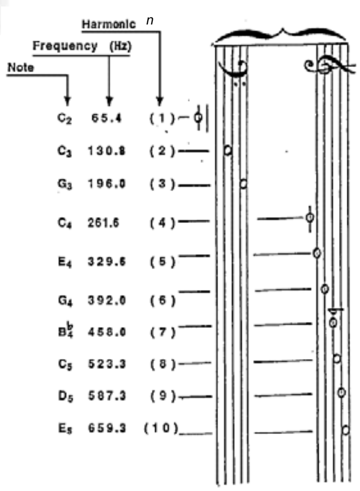


Ref.: [5], [6]

Harmoniske overtoner

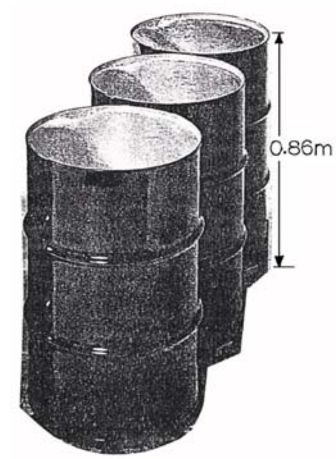
Frekvenser:

$f_n = nf_0$

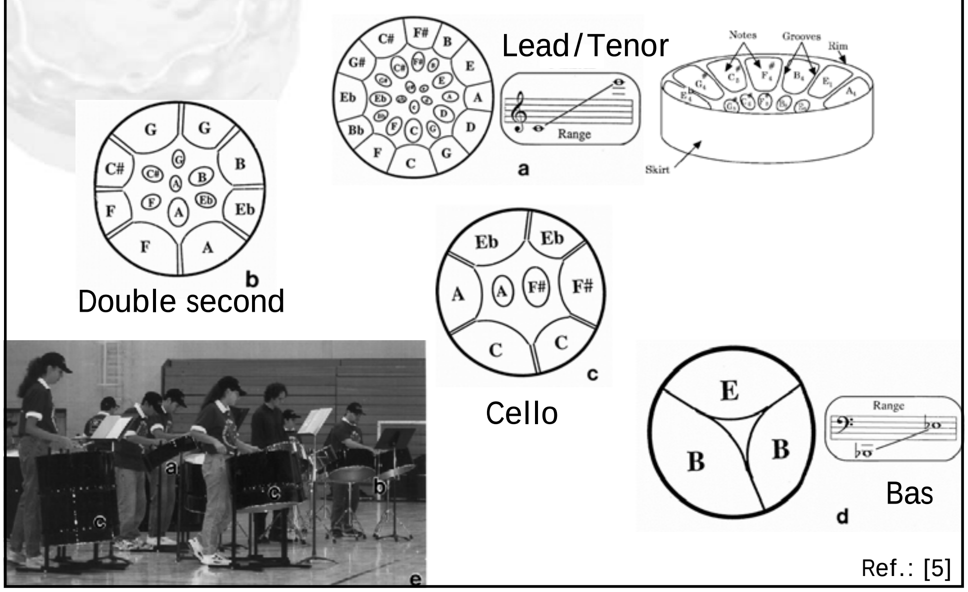


Ref.: [5]

Steelpan

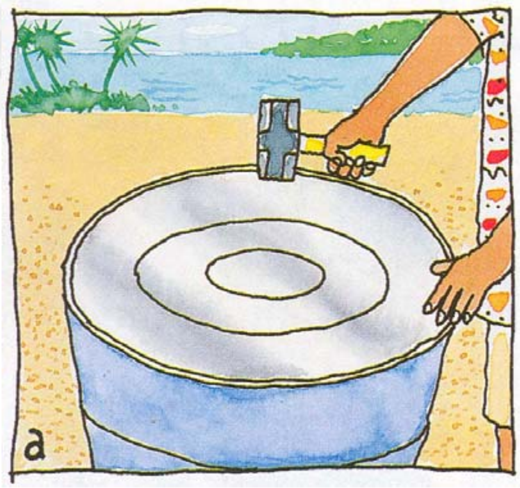


Forskellige typer



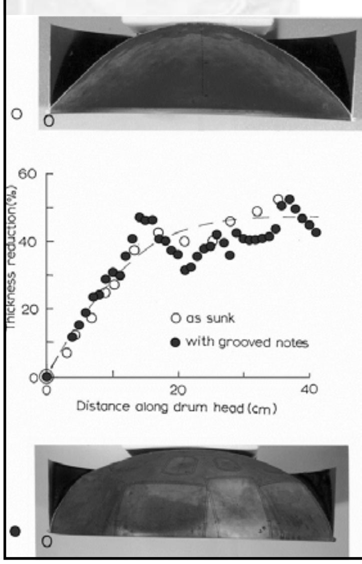
Ref.: [5]

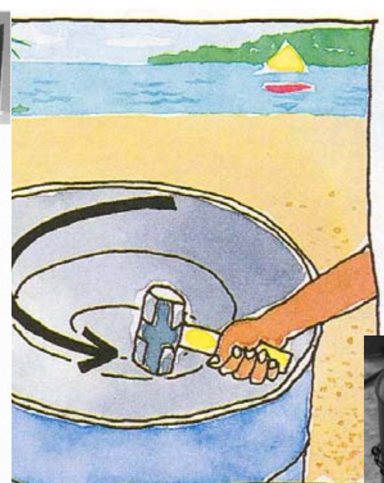
1. Start



Ref.: [4]

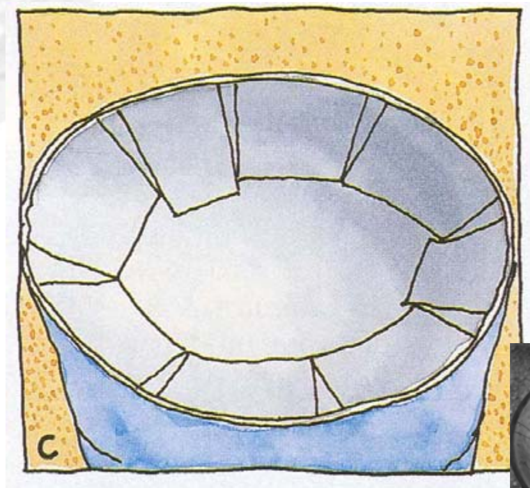
2. Sænkning

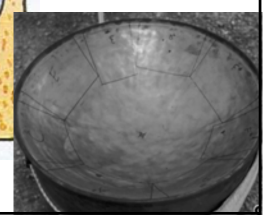




Ref.: [4], [5]

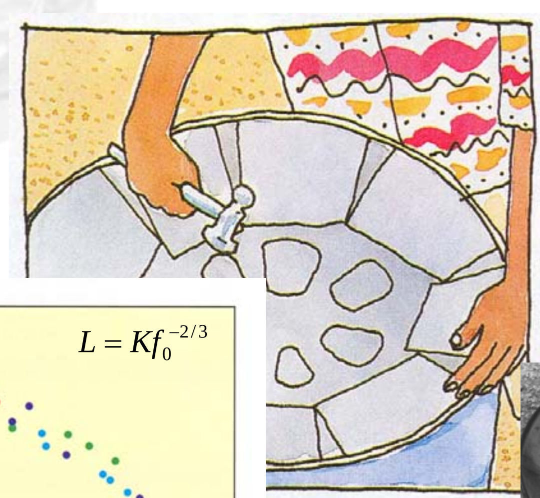
3. Nodeplacering

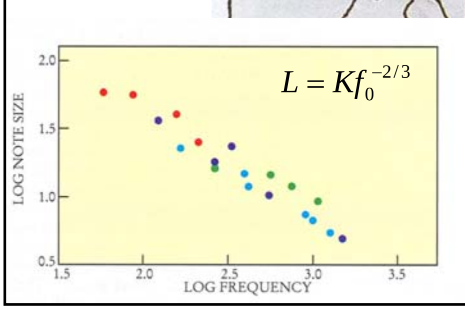




Ref.: [4], [5]

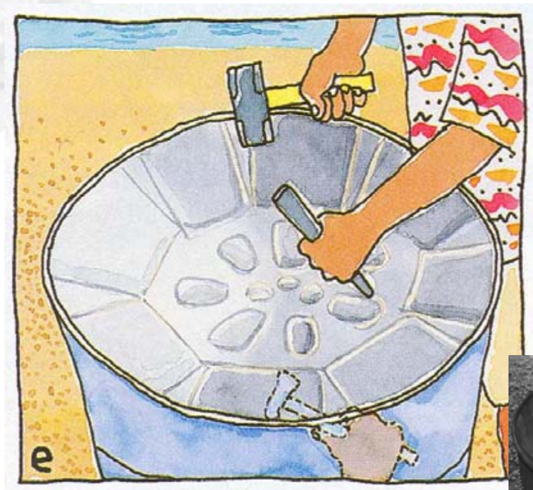
4. Nodeformning





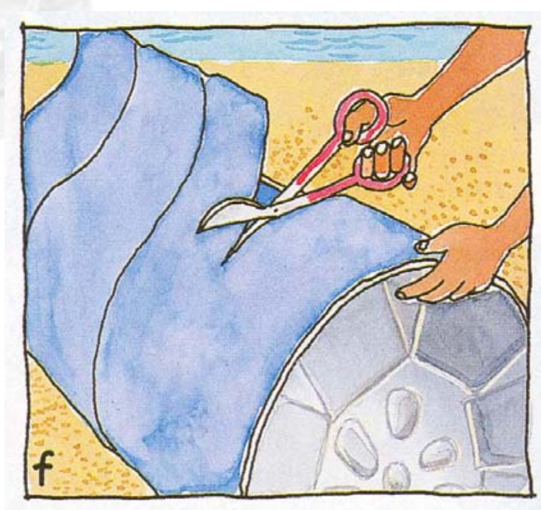
Ref.: [3], [4], [5]

5. Nodeafgrænsning



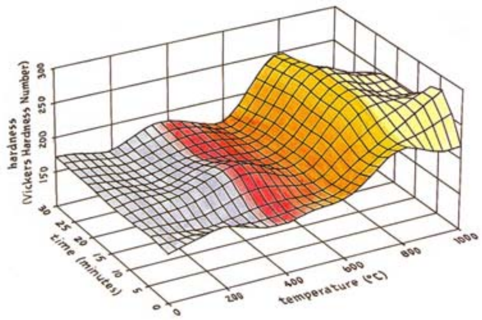
Ref.: [4], [5]

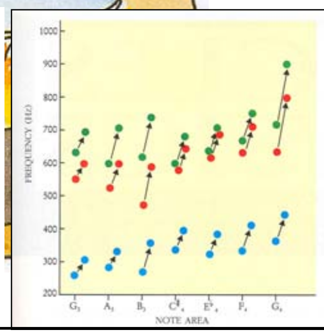
6. Skørtetilpasning



Ref.: [4]

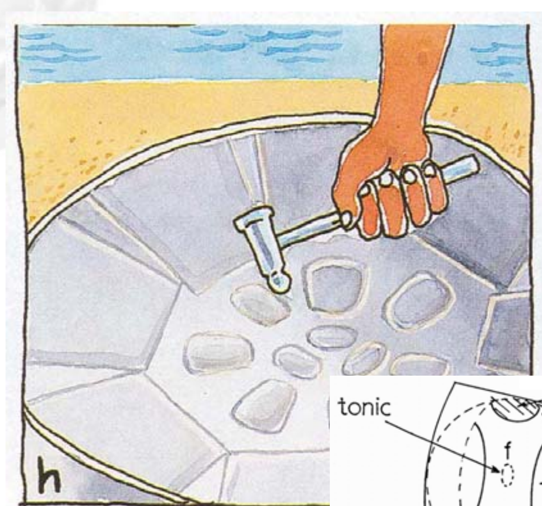
7. Varmebehandling

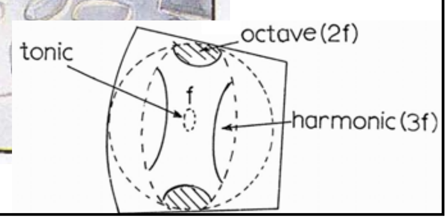




Ref: [3], [4]

8. Stemning

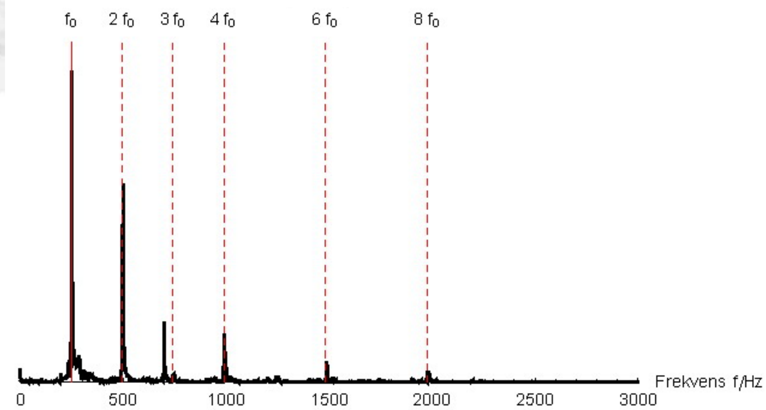




Ref.: [4], [5]

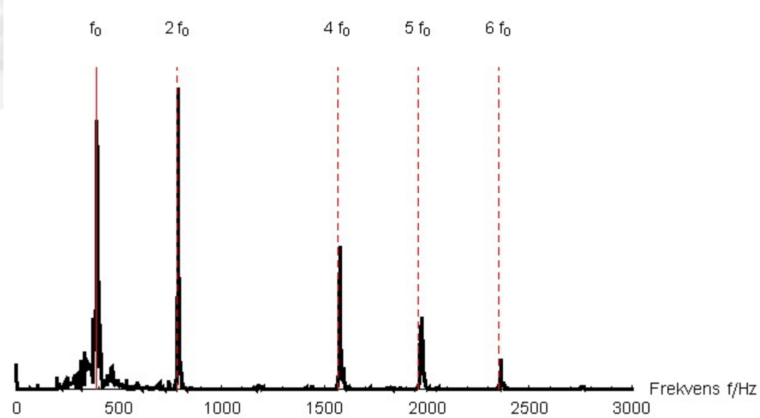
Lydspektre

H_3 = 246,9 Hz



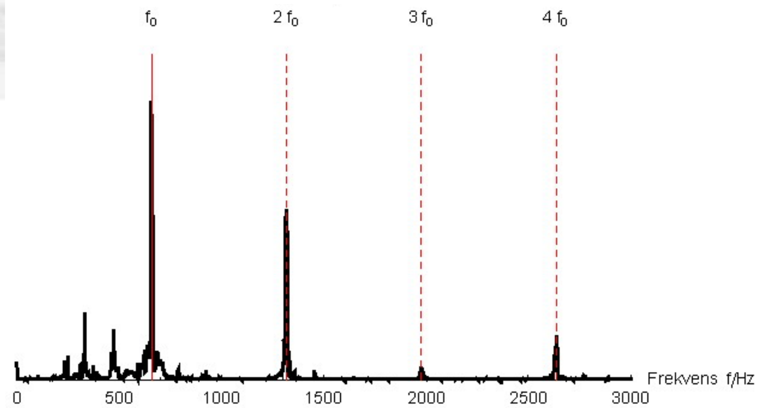
Lydspektre

G_4 = 392,0 Hz



Lydspektre

E_5 = 659,3 Hz



Lydspektre

De mange harmoniske overtoner stammer fra tre fysiske fænomener:

• Lydudstråling fra højere ordens normalsvingninger, som er blevet harmonisk stemt.

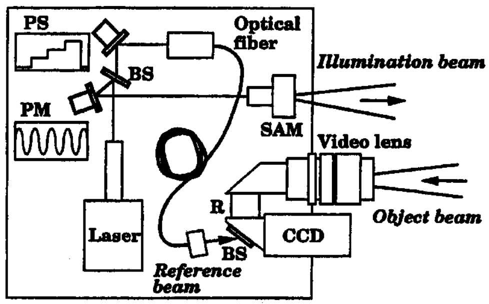
• Lydudstråling fra nabonoder, som står i et harmonisk forhold til den anslåede node (sympatisk vibration).

• Ikke-lineær bevægelse af nodeområdet ved grundfrekvensen.

Ref.: [1]

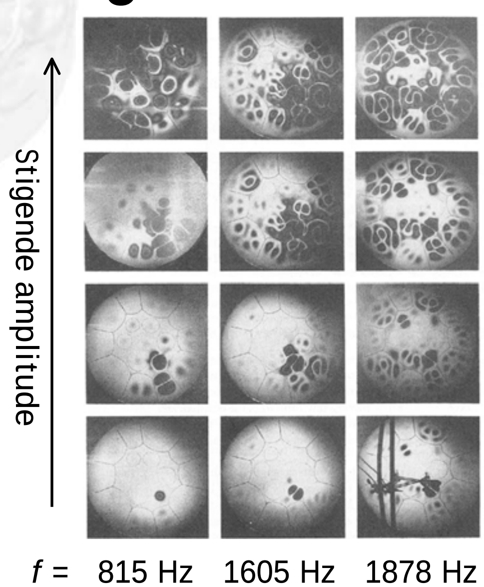
Holografisk afbildning

Den ikke-lineære kobling mellem nodeområder kan studeres vha. holografisk interferometri.



Ref.: [2]

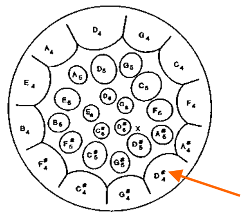
Kobling mellem noder



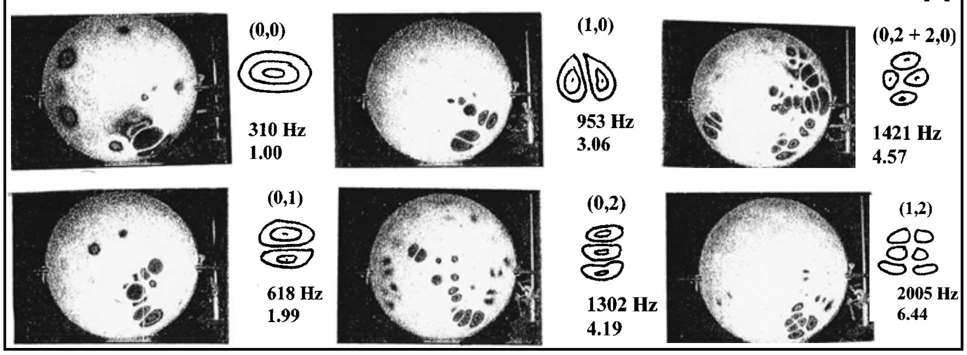
Ref.: [1]

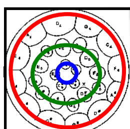
Normalsvingninger

Steelpan design:



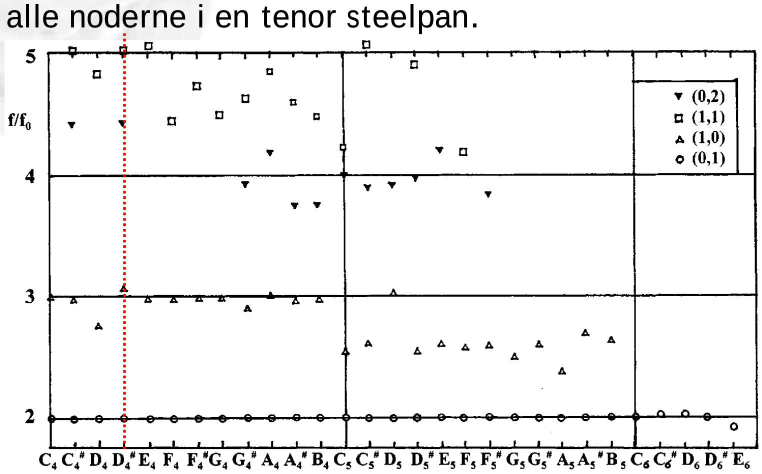
Ref.: [2]





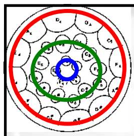
Harmonisk stemning

Oversigt over de lavfrekvente normalsvingninger i alle noderne i en tenor steelpan.

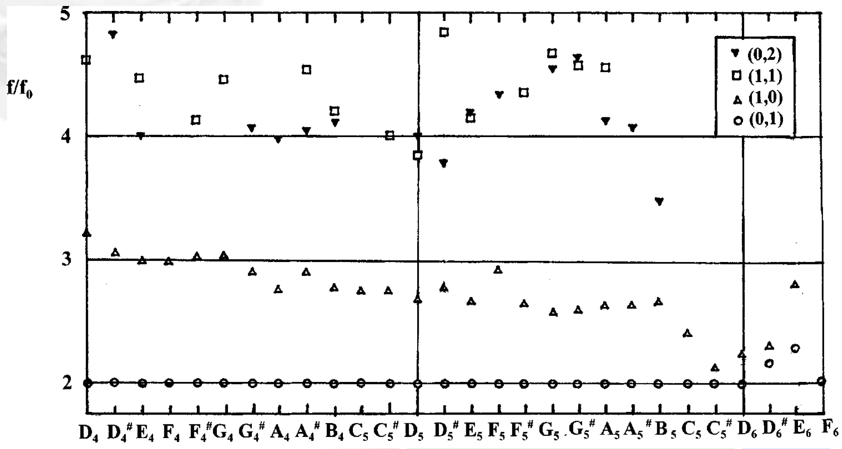


Steelpan lavet af Felix Rohner

Ref.: [2]



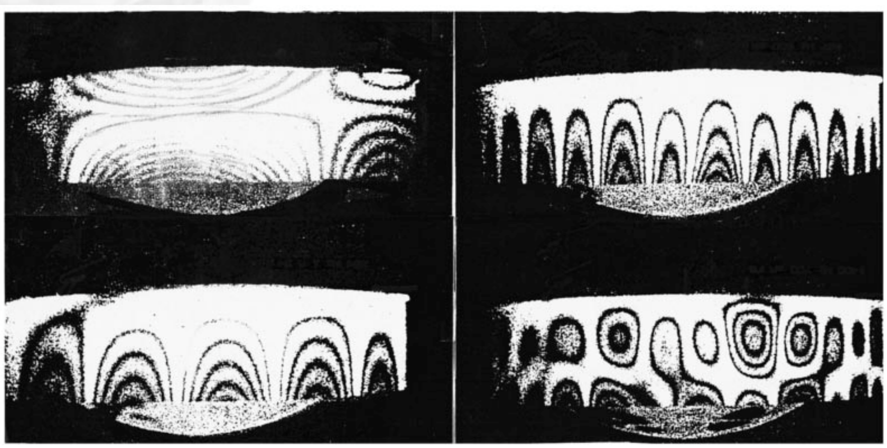
Harmonisk stemning



Tenor steelpan lavet af Clifford Alexis

Ref.: [2]

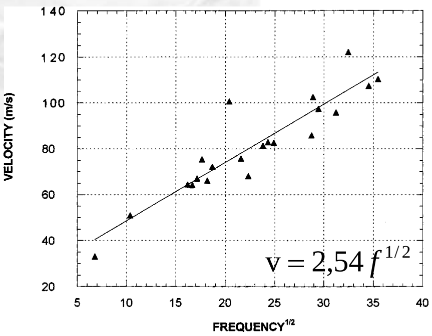
Stående bølger i skørtet



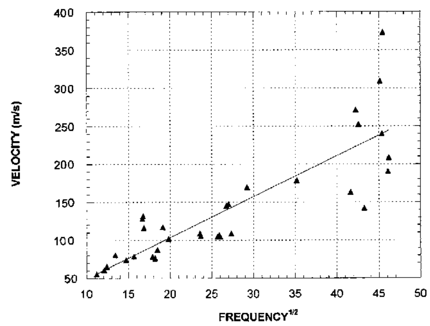
Tenor steelpan lavet af Clifford Alexis

Ref.: [2]

Stående bølger i skørtet



Clifford Alexis' steelpan



Felix Rohners steelpan

For flad, tynd plade: $v = \left(\frac{1{,}8hE}{\rho(1-\nu^2)} f \right)^{1/2} = 2{,}86\, f^{1/2}$

Ref.: [2]

Demonstration

The Roaring Lion: "Trinidad, the Land of Calypso"

på double tenor steelpans

Referencer

[1] Fletcher, Rossing: The Physics of Musical Instruments, 2. udgave, Springer, New York, 1998.

[2] Rossing, Hansen, Hampton (2000): Vibrational mode shapes in Caribbean steelpans. I. Tenor and double second, *J. Acoust. Soc. Am.*, **108**(2), 803-811

[3] Rossing, Hampton, Hansen (1996): Musik from Oil Drums: The Acoustics of the Steel Pan, *Physics Today*, **49**(3), 24-29

[4] Murr, Ferreyra (2000): Connecting Materials Science and Music in Steel Drums, *American Scientist*, **88**(1), 38-45

[5] Murr, Ferreyra *et al.* (1999): Materials science and metallurgy of the Caribbean steel drum. Part I. Fabrication, deformation phenomena and acoustic fundamentals. *J. Mater. Sci.*, **34**, 967-979

[6] Math and Music: A Primer. (Om nodeskalaen) www.members.cox.net/mathmistakes/music.htm

[7] Kreyszig: Advanced Engineering Mathematics, 8. udgave, John Wiley & Sons, kapitel 11.

… tak for opmærksomheden